Close Wed: HW_5A,5B,5C (7.1,7.2,7.3)

Office Hours: 1:30-3:00 in Smith 309

Integrate

$$\int \sin^3(x)\cos^5(x)dx$$

<u>7.2 Trig Integrals</u> (*continued*)

Entry Task: Fill in the blanks

Square Identities
$\sin^2(x) = 1 -$
$\cos^2(x) = 1 -$
$\tan^2(x) =$
$\sec^2(x) =$
Half Angle Identities
$\sin^2(x) = \frac{1}{2}$
$\cos^2(x) = \frac{1}{2}$
$\sin(x)\cos(x) = \frac{1}{2}$

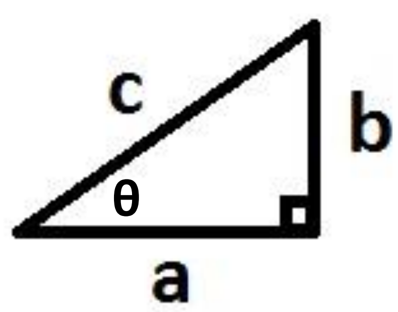
What are these in terms of a, b, and c?

sin(θ) =

cos(θ) =

tan(θ) =

sec(θ) =



Integrals involving sin(x) and cos(x)

Case 1 (*cos(x) or sin(x) has odd power*)

- Separate one from the odd power.
  (*i.e.* pull out one sin(x) or cos(x))
- Use $$\sin^2(x) = 1 - \cos^2(x)$$ $$\cos^2(x) = 1 - \sin^2(x)$$
  (get rest in term of the other)
- Use u-substitution.

Case 2 (*both sin(x),cos(x) even powers*)

- Use
$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$
  over and over again until you can integrate.

Example:

$$\int \cos^4(x)\,dx$$

Integrals involving tan(x) and sec(x)

Case 3 (sec*(x) has an even power*)

- Separate out $\sec^2(x)$
- Use $\sec^2(x) = 1 + \tan^2(x)$
  (get rest in terms of $\tan(x)$)
- Use $u = \tan(x)$

Example: $\int \tan^5(\mathrm{x})\sec^4(\mathrm{x})dx$

Case 4 (tan*(x) has an odd power*)

- Separate out $\sec(x)\tan(x)$
- Use $\tan^2(x) = \sec^2(x) - 1$
  (get rest in terms of $\sec(x)$)
- Use $u = \sec(x)$

Example: $\int \tan^3(\mathrm{x})\sec(\mathrm{x})dx$

And if you've tried the four cases and are stuck, here are things to try:

1. Rewrite in terms of sin(x) and cos(x).
2. Rewrite in terms of sec(x) and tan(x).
3. Try using trig identities.

There are still a few "holes".
Namely, when there is one tan(x) or an odd power on sec(x). You can quote these (proof in the book):

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(\mathrm{x}) + \tan(\mathrm{x})| + C$$

7.3 Trigonometric Substitution

Goal: Develop a method to evaluate integrals involving expressions of the form $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$

CASE	**SUBSTITUTION**
$a^2 - x^2$	$x = a\sin(\theta)$, $-\pi/2 \le \theta \le \pi/2$
$a^2 + x^2$	$x = a\tan(\theta)$, $-\pi/2 < \theta < \pi/2$
$x^2 - a^2$	$x = a\sec(\theta)$, $0 \le \theta < \pi/2$, $\pi \le \theta < 3\pi/2$

Example:

1. $\int \frac{x^3}{\sqrt{4 - x^2}}\,dx$

Trigonometric Substitution Method:

A) Substitute, don’t forget dx = ??dθ.
   Simplify (eliminate root)
B) Use 7.2 methods for trig integrals.
C) Draw a triangle and return to x.

2. $\int \sqrt{9 + x^2}dx$

3. $\int \frac{\sqrt{x^2 - 16}}{x}\, dx$

Completing the Square:

$$\sqrt{ax^2 + \boldsymbol{bx} + c}$$

If you encounter a "**middle term**" (like **bx** above), then complete the square.

Example: $\sqrt{64 - 24x - 4x^2}$

i) Factor out the "a".

$$\sqrt{4(16 - 6x - x^2)} = 2\sqrt{16 - 6x - x^2}$$

ii) Add/subtract "half-middle squared"

Half of middle = (-6)/2 = -3

Squared = $(-3)^2$ = 9

$$2\sqrt{16 + 9 - 9 - 6x - x^2}$$

iii) Factor the perfect square

$$2\sqrt{25 - (x + 3)^2}$$

iv) **Check your work!!!!!**

Example:

$$\int \frac{x}{\sqrt{64 - 24x - 4x^2}}\,dx$$